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Random Investment Method in the Risk Management Problem

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Abstract: The article discusses the method of random investment in the problem of risk management. An algorithm for calculating the optimal distribution for a limited sample is formulated. An example of calculating the optimal distribution is given.
Keywords: portfolio, profitability, risk, investment, sample.

Introduction

Let's consider a sample constituted by vectors $V = (R_i)_{i=1}^N$ of risky assets. We will define a portfolio as a vector $x = (x_i)_{i=1}^n$, such that $\sum_{i=1}^n x_i = 1$, n – number of asset types. The profitability of a portfolio is a random variable: $r = (R, x)$. According to Markowitz's theory [1], the quality of a portfolio is determined by two parameters: average return Er and variance Dr. The problem of computing the optimal portfolio involves a multi-objective (vector) criterion. Essentially, the portfolio ought to be selected to maximize the average return while minimizing the variance (risk). An overview of methods for solving this problem is presented in [2,3]. In addition to variance, there are other functionals for evaluating the quality of a portfolio [4-6]. The solution to a problem with a vector criterion is typically regarded as a set of non-dominated strategies, known as the Pareto set. One approach to computing Pareto-optimal portfolios involves scalarizing the vector criterion. The expected value and variance are functions of the portfolio: $Er = (\overline{R}, x), Dr = (\overline{C}x, x)$. To calculate these values, one has to know the probability measure, yet this measure is generally unknown. Alternatively, a training sample can be adopted to compute the sample mean vector $\overline{R}$ and the sample covariance matrix $\overline{C}$. Consequently, both the evaluation criterion and the optimal portfolio are contingent upon the sample. A significant issue at hand is the calculation of an optimal portfolio that exhibits stability against sample variations. References [7, 8] propose an algorithm for constructing an optimal portfolio through cluster analysis. Reference [9] demonstrates an algorithm for establishing an optimal portfolio by leveraging a confidence set. Reference [10] introduces robust algorithms for formulating an optimal portfolio based on the MCD (Minimal Covariance Determinant) method and the Wasserstein metric. Moreover, reference [11] demonstrates the effectiveness of the MCD algorithm when dealing with contaminated samples.

In this article, random investment, defined by the random variable $\xi \in \{1,\ldots,n\}$ is discussed. Random investment corresponds to random return r_ξ. In the case of multiple investments, we can consider the average return as the conditional mathematical expectation: $r = E(r_\xi / R) = (R,P)$, where $P = (P_i)_{i=1}^n, P_i = P(\xi = i)$. In this regard, it should be noted that the portfolio quality indicator in Markowitz's diversification theory coincides with the quality indicator for multiple random investments, provided that short-selling is prohibited.

Let's consider the VaR (Value at Risk) criterion, for which the problem of optimal random investment takes the following form:

$$\begin{aligned} &\max \alpha, \\ &P((R,P) \geq \alpha) \geq \beta, \\ &P_i \geq 0, \sum_i P_i = 1. \end{aligned} \tag{1}$$

To use this criterion, it is necessary to know the distribution law of the random vector R. For example, for the normal distribution law, problem (1) takes the following form:

$$\begin{aligned} &\max\left((m,P) + \Phi^{-1}(1-\beta)\sqrt{(CP,P)}\right), \\ &P_i \geq 0, \sum_i P_i = 1. \end{aligned} \tag{2}$$

In problem (2), m is the vector of mean values, and C is the covariance matrix. These parameters of the normal distribution law can be estimated from a sample. To estimate these parameters, alternative approaches such as robust optimization methods or forecasting techniques can be employed.

The main problem is that the vector R may have an unknown distribution law that differs from the normal distribution. In this case, parametric statistics methods can be used when the sample is limited, or machine learning methods [12] can be used when the sample is unlimited.

Suppose that the sample size is limited. We will consider the empirical probability measure generated by the sample $V=\{R_1,...,R_N\}$, namely, $P_V(A)=\frac{1}{N}\sum_{R\in V} I_{\{R\in A\}}=\frac{N_A}{N}$. For the empirical measure, problem (1) takes the following form:

$$\begin{aligned} &\max \alpha(A), \\ &A\subseteq V, \frac{N_A}{N}\geq\beta, \\ &(R,P)\geq\alpha, R\in A, \\ &P_i\geq 0, \sum_i P_i=1. \end{aligned} \tag{3}$$

For a fixed set A, problem (3) is a linear programming problem, and the main problem lies in the selection of the optimal set A^* such that $\alpha^*(A^*)\geq\alpha^*(A)$. An exact solution can be found among the subsets of the original sample with the number of elements $N_A=\begin{cases} N\beta, & N\beta - \text{integer} \\ [N\beta]+1, & \text{otherwise} \end{cases}$. The number of linear programming problems that need to be solved in this case is equal to the number of combinations of N elements taken N_A at a time, which can be quite large. Therefore, a monotonic algorithm is presented below, which allows us to find an acceptable solution that is not always the optimal one. This algorithm is a modification of the MCD algorithm, which has been successfully used in computing robust estimates of the vector of mean values and the covariance matrix.

Algorithm.

1. Initialization. A subset A of the sample V, containing N_A elements, is randomly selected, and $\alpha = 0$ is assumed.

2. Problem (3) is solved for the subset A. If $\alpha < \alpha^*(A)$, then $\alpha = \alpha^*(A)$. Using the obtained vector $P^*(A)$, the elements of the sample need to be ordered. Specifically, a permutation should be formed on the set of indices π : $\left(P^*(A), R_{\pi(i)}\right) \geq \left(P^*(A), R_{\pi(j)}\right)$ if $i < j$. Then, form the set $A = \{R_{\pi(1)}, \ldots, R_{\pi(N_A)}\}$ and repeat step 2; otherwise, STOP.

Remark 1. This remark pertains to single-time random investment. For single-time investment, using the conditional average return as an indicator of investment quality seems unnatural. It is more reasonable to measure the quality of an investment directly by the random variable r_ξ. This will lead to a change in the problem discussed in the article, which will now take the following form:

$$max\, \alpha, \tag{4}$$

$$\sum_{i=1}^{n} \nu_i(\alpha) p_i \geq \beta$$

$$\sum_{i-1}^{n} p_i = 1, p_i \geq 0.$$

In (4), $\nu_i(\alpha) = \frac{1}{N}\sum_{i=1}^{N} I_{\{r_{i,j} \geq \alpha\}}$, $r_{i,j}$ is the i-th coordinate of the j-th vector of the sample. Without going into details, it should be noted that the solution to this problem is a probability distribution concentrated at one atom, that is, one should invest in a single asset.

Remark 2. In many optimal investment problems, for example, when calculating option prices, a risk-free asset is presented among risky assets. Let's add a risk-free asset with a fixed return α to the set of assets available to the investor. Investing in the risk-free asset at time t yields a return of α. Investing in a risky asset at time t yields a return of r_ξ. At time t, the investor knows the values of P_t and the values of $\beta_t = (P_t, \theta_t)$ for which $r_\xi \geq \alpha$. In this formulation, the optimal investment problem turns into a game with a one-armed bandit. It is shown in [13] that the optimal behavior of the investor is a mixed strategy, that is, with a probability of $1-\beta_t$, the investment is made in the risk - free asset; with a probability of β_t, the investment is made in the risky asset.

Example.

Let's consider a sample consisting of 298 elements, and the number of asset types is 5.

Table 1 presents the solution to problem (3) when $N_A = 250$. Specifically, three iterations of the algorithm that lead to the solution of the problem are given.

Table 1.

Iteration	α	p_1	p_2	p_3	p_4	p_5
1	0.93	0.437	0.073	0.003	0.337	0.150
2	0.97	0.370	0.083	0.013	0.349	0.185
3	0.97	0.386	0.079	0.018	0.353	0.164

Conclusion

The article discusses a risk reduction technique that uses a random investment. It has been established that this investment is expedient in case of multiple investments (remark 1). For a multiple random investment, the problem of VaR optimization for the empirical distribution law is approximately solved. We believe that for this investment it is possible to set the task of VaR-online optimization. In this article, a risk-reduction technique using random investment is discussed. It has been established that such investment is advisable for multiple investments (remark 1). For multiple random investments, the problem of VaR-optimization for the empirical distribution law has been approximately solved. We assume that it is possible to formulate the problem of VaR-online optimization for this investment.

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